Units and Calculations – Using Decibels

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A task that pops up all the time when you are attending spectrum management meetings is the need to do a quick calculation in your head, to add quantitative validity to the discussion. In a more complicated case you might have to sit down and work something out on paper during a lunch break, but you won't have the luxury of several days to think about it. For these reasons, it's very useful to know how to get "good enough" answers quickly.

LOGARITHMIC SCALING

A logarithmic factor of 10 is called a *Bel*, in honor of Alexander Graham Bell. If a quantity is 10^B, then B is its representation in Bels, using the base-10 logarithm rule you learned in school:

$$\mathbf{\log_{10} (10^B) = B \quad [Bels]}$$

Thus the number 100 is 2 Bels, because $\log 10^2 = 2$. Negative Bel values represent values less than one, so $0.01 = 1/100 = 10^{-2}$ is -2 Bels, because $\log_{10} 10^{-2} = -2$.

While Bels are rarely used, the *decibel*, which is 1/10th of a Bel, is, by contrast, the lingua franca of the engineering community. The abbreviation for the decibel is dB, and the equation relating a quantity D to itself in dB units has an extra '10' in it, such that

$$\mathbf{D = 10 \log_{10} 10^D \quad [dB].}$$

To return to our examples, the number 100 is 20 decibels, because $10 \log_{10} 10^2 = 20$, and $0.01 = 1/100$ is -20 dB, because $10 \log_{10} 10^{-2} = -20$.

Perhaps we should keep the **B** in 'decibel' capitalized, but the conventional usage is to spell it in lower case. The honorific capitalization does remain in dB, much to the confusion of typists and word processing software.

NUMERICAL INTERLUDE

There's a good reason for using dB – you can calculate to 1 % with almost no memorization. Factors of 10 are fairly obvious:

Value:	1/1000	1/100	1/10	1	10	100	…	1,000,000,000
In dB:	-30	-20	- 10	0	10	20		90

If you can just remember that conversion of a power of 10 to dB gives

“*Number* of zeros, with a zero after it, negative sign if less than 1”

you are home free.

The real advantage comes from two numerical quirks:

1) A factor of 2 is 3.0102999566 dB, but this differs from 3 dB by only 0.34 %.
2) The square root of 10 is exactly 5 dB, but is larger than π by only 0.66 %.

With just these two facts, it is easy to assemble the following three tables, which contain all the integer values for dB, correct within better than 1 %:

Factor:	1	2	4	8
In dB:	0	3	6	9

Factor:	10	5	5/2	5/4
In dB:	10	7	4	1

Factor:	$\pi/2$	π	2π	4π
In dB:	2	5	8	11

If you remember where the table entries come from, you only have to memorize that a factor of 2 is 3 dB and a factor of π is 5 dB. You can look at 77 dB, and realize immediately that it is a factor 2 smaller than 10^8, or 50,000,000. When you want the isotropic aperture $\lambda^2/4\pi$ for a wavelength λ of 1 m, you know immediately that it is 11 dB less than one square meter, or 0.080 square meters. Both answers are correct to better than 1 %.

It is very useful to know conversions for small changes:

1/10 dB is close to +2 % (actually, 2.3 %)

+1 dB is close to +25 % (actually 25.9 %)

-1 dB is close to –20 % (actually -20.6 %).

So if a room-temperature wave-guide has a 0.1 dB loss, it will add 2.3 % of room temperature, or 6.7 K, to the system temperature of a receiver. If it is in the output path of a megawatt radar transmitter it will absorb 23 kW.

Finally, an occasionally useful link to optical astronomy:

A difference of +1 stellar magnitude is exactly -4 dB.

Stellar magnitudes arose originally as the smallest difference in apparent brightness discernable by the human eye, with fainter stars having higher magnitudes. This was later quantified by equating 5 magnitudes to the difference in brightness for two identical stars differing in distance by a factor of 10, and so in apparent brightness by a factor of 1/100. Since a factor of 1/100 is -20 dB, and this corresponds to +5 magnitudes, then +1 magnitude is 1/5 of this, or -4 dB.

What about the Units?

Quantities expressed in dB are always ratios, and hence pure numbers. Thus, for the case of power, we have P (dB) = 10 $\log_{10}$ (P / P_0), where there are several options for P_0 – Watts, milliwatts, etc. The *unit* is therefore commonly appended to dB, which in this case becomes dBW, dBm, etc. Note here the notation dBm for the milliwatt unit, though dBm^2 would be used in the case of square metres. Nor is the dB notation limited to power, as for example:

– Bandwidth B:	10 MHz	⇔ 70 dBHz
– Time τ:	2000 seconds	⇔ 33 dBs

Moreover seconds * Hz gives a pure number, so

– $\sqrt{(B\,\tau)}$: (70 dBHz + 33 dBs) / 2 = 51.5 dB

Useful Definitions

In the expression for power, $P = k\,T\,B$

T is the absolute temperature in Kelvin degrees
B is the bandwidth
k is Boltzman's constant, 1.38 10^{-23} Joules/Kelvin (-228.6 dBW/Hz/K)

Hence at room temperature (290 K),

$$kT\,dB = -204.0 \text{ dBW/Hz.}$$

Consider next Power Flux Density (PFD), which is the radiated power passing through a given area, and so often has units of W/m^2. The Spectral Power Flux Density is then the PFD per unit bandwidth, or $W/m^2/Hz$. Hence

1 Jansky is 10^{-26} $W/m^2/Hz$ (sum of both polarizations) ⇔ -260 $dBW/m^2/Hz$

The *Isotropic Aperture* (unity gain in all directions) at wavelength λ is $A_i = \lambda^2 / 4\pi$ [m^2], which is the area of a circle with a circumference of λ. The isotropic aperture drops off rapidly with increasing frequency:

Wavelength	Isotropic Aperture
1 m	-11 dBm^2
1 mm	-71 dBm^2

Effective Aperture with Gain G is then $A_e = G\ A_i = G\ \lambda^2 / 4\ \pi$ [m^2].

Let us work a few examples

For Tsys: if you know that a room temperature of 290 K is –204 dBW/Hz, what is a Tsys = 29 K in these units? Answer -214 dBW/Hz.

For A_i: You are observing at 20 cm. What is your isotropic aperture in dBm^2? -25 dBm^2.

Radiometer Equation: You observe for 2000 seconds with a bandwidth of 10 MHz. What is your ΔT / Tsys = $1 / \sqrt{(B\ \tau)}$? Answer -51.5 dB.

What SPFD arriving in an isotropic sidelobe equals this noise power? That is, what SPFD radiated at 20 cm matches the k ΔT power indicated by the radiometer equation as being equal to the sensitivity of a 29 K Tsys receiver in a 2000 s integration? Answer -241.5 dBW/m^2/Hz.
Hint: One needs much more radiated power to compensate for the loss into an antenna, so the answer is an amalgam of the first three examples, namely the SPFD = Tsys * (ΔT / Tsys) / (isotropic aperture), which is numerically {-214 -(-25) -51.5 dB}.